Exercice 1

*/ $\int_2^1 (2x+1)\,dx = \left[x^2+x\right]_2^1$

$= (1+1)-(4+2) = 2-6 = -4$

*/ $\int_1^2 (x^2-2x+3)dx = \left[\frac{x^3}{3}-x^2+3x\right]_1^2$

$= \left(\frac{8}{3}-4+6\right)-\left(\frac{1}{3}-1+3\right) = \frac{14}{3}-\frac{7}{3} = \frac{7}{3}$

*/ $\int_0^1 \frac{2x+1}{(x^2+x+1)^2}\,dx = \int_0^1 \frac{(x^2+x+1)'}{(x^2+x+1)^2}dx$

$= \left[\frac{-1}{x^2+x+1}\right]_0^1 = -\frac{1}{3}+1 = \frac{2}{3}$

*/ $\int_0^{\sqrt{2}} \frac{x}{\sqrt{x^2+1}}\,dx = \int_0^{\sqrt{2}} \frac{(x^2+1)'}{2\sqrt{x^2+1}}dx$

$= \left[\sqrt{x^2+1}\right]_0^{\sqrt{2}}$

$= \sqrt{3}-1$

*/ $\int_1^2 \frac{x^2+x+1}{x}dx = \int_1^2 \left(x+1+\frac{1}{x}\right)dx$

$= \left[\frac{x^2}{2}+x+\ln x\right]_1^2 = \left(\frac{4}{2}+2+\ln 2\right)-\left(\frac{1}{2}+1+\ln 1\right)$

$= 4+\ln 2-\frac{3}{2} = \frac{5}{2}+\ln 2$

*/ $\int_0^2 \left(1-|x-1|\right)dx = I$

On a:

x	$-\infty$	1	$+\infty$
$x-1$	$-$	0	$+$
$\lvert x-1\rvert$	$-(x-1)$		$x-1$
$1-\lvert x-1\rvert$	x	0	$2-x$

D'où :

$I = \int_0^1 \left(1-|x-1|\right)dx + \int_1^2 \left(1-|x-1|\right)dx$

$= \int_0^1 x\,dx + \int_1^2 (2-x)\,dx$

$= \left[\frac{x^2}{2}\right]_0^1 + \left[2x-\frac{x^2}{2}\right]_1^2$

$= \frac{1}{2}+\left[\left(4-\frac{4}{2}\right)-\left(2-\frac{1}{2}\right)\right]$

$= \frac{1}{2}+2-2+\frac{1}{2} = 1$

*/ $\int_2^3 \frac{1}{3x+2}\,dx = \int_2^3 \frac{3}{3(3x+2)}\,dx$

$= \frac{1}{3}\int_2^3 \frac{(3x+2)'}{(3x+2)}\,dx$

$= \frac{1}{3}\left[\ln(3x+2)\right]_2^3 = \frac{1}{3}\left[\ln 11-\ln 8\right]$

$= \frac{1}{3}\ln\frac{11}{8}$

*/ $\int_0^1 e^{2x}dx = \left[\frac{e^{2x}}{2}\right]_0^1 = \frac{e^2-1}{2}$

*/ $\int_1^2 \frac{x}{x+1}dx = \int_1^2 \frac{x+1-1}{x+1}dx$

$= \int_1^2 \left(\frac{x+1}{x+1}-\frac{1}{x+1}\right)dx = \int_1^2 \left(1-\frac{1}{x+1}\right)dx$

$= \left[x-\ln(x+1)\right]_1^2 = \left[(2-\ln 3)-(1-\ln 2)\right]$

$= 2-\ln 3-1+\ln 2 = 1+\ln\frac{2}{3}$

*/ $\int_1^e \frac{\ln x}{x}dx = \int_1^e \frac{1}{x}.\ln x\,dx$

$= \int_1^e (\ln x)'.\ln x\,dx = \left[\frac{1}{2}\ln x^2\right]_1^e$

$= \frac{1}{2}\ln^2 e-0 = \frac{1}{2}$